Solutions

3.3: Higher-Order Linear, Homogeneous Equations with Constant Coefficients
Factoring Techniques

Equations of Quadratic Type Use substitution to solve the following.

- $x^4 + 14x^2 + 45 = 0$
- $(x^2 + x)^2 + 18(x^2 + x) + 72 = 0$

- $0 = x^4 + 14x^2 + 45 = (x^2)^2 + 14(x^2) + 45 = (x^2+9)(x^2+5)$

  $\Rightarrow x = \pm 3i, \pm\sqrt{5}\, i$

- $u = x^2 + x \Rightarrow 0 = u^2 + 18u + 72 = (u+6)(u+12) \Rightarrow u = -6, -12$

  So $x^2 + x = -6$ or -12,
  - $x^2 + x + 6 = 0 \Rightarrow \frac{-1 \pm \sqrt{1-24}}{2} = -\frac{1}{2} \pm i\sqrt{23/4}$
  - $x^2 + x + 12 = 0 \Rightarrow \frac{-1 \pm \sqrt{1-46}}{2} = -\frac{1}{2} \pm i\sqrt{47/4}$

Factor by Grouping Solve the following.

- $x^3 - x^2 + 4x - 4 = 0$
- $2r^2 - 7r + 3 = 0$

- $0 = x^3 - x^2 + 4x - 4 = x^2(x-1) + 4(x-1) = (x^2+4)(x-1) \Rightarrow x = 1, \pm 2i$

- $0 = 2r^2 - 7r + 3 \Rightarrow \begin{cases} 2 \cdot 3 = 6 \\ b = -7 \end{cases}$ Let $m = -6, n = -1$ so that $m \cdot n = 6$ and $m + n = -7$.

Then $2r^2 - 7r + 3 = 2r^2 - 6r - r + 3 = 2r(r-3) - (r-3) = (2r-1)(r-3)$

$\Rightarrow r = \frac{1}{2}, 3$.

Long Division Solve the equation $x^3 - x^2 + 4x - 4 = 0$ given that $x = 1$ is one of the solutions.

$$\begin{array}{r|rrrr} 1 & 1 & -1 & 4 & -4 \\ & & 1 & 0 & 4 \\ \hline & 1 & 0 & 4 & 0 \end{array} \Rightarrow x^2 + 4$$

So

$x^3 - x^2 + 4x - 4$

$= (x-1)(x^2+4)$

$\Rightarrow x = 1, \pm 2i$